Fill in the blank with the value that makes the statement true, then write a formal proof of the resulting statement. SCORE: _____/9 PTS

"For all integers n, if $n \mod 3 = 2$, then $(n^2 - 6) \mod 3 = 1$."

UNLESS OTHERWISE

Let n be a particular but arbitrary chosen integer such that $n \mod 3 = 2$.

So,
$$n = 3q + 2$$
 for some $q \in \mathbb{Z}$ by definition of mod.
So, $n^2 - 6 = 9q^2 + 12q - 2 = 3(3q^2 + 4q - 1) + 1$ where $3q^2 + 4q - 1 \in \mathbb{Z}$ by closure of \mathbb{Z} under \times and $+$.

So,
$$(n^2 - 6) \mod 3 = 1$$
 by definition of mod.

PROOF:

Find the values of (-39) div 11 and (-39) mod 11. SCORE: 4 PTS Justify your answers VERY briefly. You do NOT need to write a proof. (-39) div 11 = -4 and (-39) mod 11 = 5

since $-39 = -4 \times 11 + 5$

Write the Ouotient Remainder Theorem symbolically. $\forall d \in \mathbf{Z}^+, \mid \exists ! q, r \in \mathbf{Z} : \mid n = dq + r \mid \land \mid 0 \le r < d \mid$

One of the following statements is true and one is false. SCORE: / 18 PTS State clearly which statement is false, show that it is false, then write a formal proof for the true statement.
[a] If the sum of two integers is odd, then exactly one of the integers is odd.[b] The set of irrational numbers is closed under multiplication.
[a] is true. There are two possible solutions, depending on whether you used contraposition or contradiction. SOLUTION 1: GRADE AGAINST ONLY I SOLUTION
CONTRAPOSITIVE: For all integers x and y , if it is not the case that exactly one of x and y is odd, then $x + y$ is not odd.
PROOF BY CONTRAPOSITION:
Let x and y be particular but arbitrary chosen integers, such that it is not the case that exactly one of x and y is odd. So, either both x and y are odd, or neither x nor y are odd.
CASE 1: Both x and y are odd So, $x = 2m + 1$ and $y = 2n + 1$ for some integers m and n by definition of odd
$(x + y = 2(m + n + 1)) \text{ where } m + n + 1 \in \mathbf{Z} \text{ by the closure of } \mathbf{Z} \text{ under } + \mathbf{Z} \text{ so, } x + y \text{ is even, by definition of even, } \mathbf{Z} \text{ so, } \mathbf{Z} \text{ where } \mathbf{Z} \text{ is even, } \mathbf{Z} \text{ so, } \mathbf{Z} \text{ and } \mathbf{Z} \text{ so, } \mathbf{Z} so, $
So, $x + y$ is not odd by Parity Property
CASE 2: Neither x nor y are odd So, both x and y are even by Parity Property
So, $x = 2m$ and $y = 2n$ for some integers m and n by definition of even
So, $x + y$ is even by definition of even So, $x + y$ is not odd by Parity Property
So, $x + y$ is not odd
Therefore, by contraposition, if the sum of two integers is odd, then exactly one of the integers is odd
(1) MUST STATE FULL SENTENCE FOR THIS POINT
LOT JUST

NOT JUST "THE STATEMENT IS TRUE"

SOLUTION 2:

PROOF BY CONTRADICTION:

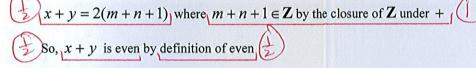
Suppose not, that is, suppose there are integers x and y such that x + y is odd,

but it is not the case that exactly one of x and y is odd.

So, either both x and y are odd, or neither x nor y are odd.

CASE 1: Both x and y are odd

So,
$$x = 2m + 1$$
 and $y = 2n + 1$ for some $m, n \in \mathbb{Z}$ by definition of odd



CASE 2: Neither x nor y are odd

So, both
$$x$$
 and y are even by Parity Property

So, $x = 2m$ and $y = 2n$ for some $m, n \in \mathbb{Z}$ by definition of even

$$x + y = 2(m + n)$$
 where $m + n \in \mathbb{Z}$ by the closure of \mathbb{Z} under $+$

So,
$$x + y$$
 is even by definition of even $x + y$ is odd and $x + y$ is even (contradiction of Parity Property)

So, x + y is odd and x + y is even (contradiction of Parity Property)

NOT JUST "THE STATEMENT

Therefore, by contradiction, if the sum of two integers is odd, then exactly one of the integers is odd []

GIVE THIS EXAMPLE TO GET

[b] is false. For example, $\sqrt{2}$ is irrational, but $\sqrt{2} \times \sqrt{2} = 2 = \frac{2}{1}$ is not irrational.